Exercise 1 – Use direct calculus to find the primitive

$$\int \left(x + \frac{1}{x^2} + \sqrt{x}\right) dx, \int \left(5x^3 - 10y^2 + 8yx + 1\right) dx (y \in \mathbb{R}), \int \frac{1-\theta^2}{\theta} d\theta,$$
$$\int t\sqrt{t^2+4} dt, \int \frac{x^4}{(5x^5-1)^3} dx, \int \left(1 + \tan^2(3x)\right) dx, \int \sin(x)\cos^3(x) dx, \int \frac{1}{\sin(2x)} dx.$$

Exercise 2 – Let $a, b \in \mathbb{R}^*$ and let F and G be the functions

$$F(x) = \int e^{ax}\cos(bx) dx, G(x) = \int e^{ax}\sin(bx) dx.$$

By means of the integration by part find the relation between F and G. Deduce there expressions.

Exercise 3 – Using the substitution, find the primitive

$$\int \frac{2}{25x^2+9} dx, \int x\cos(x^2+1) dx, \int \frac{e^{3x}}{1+e^x} dx,$$
$$\frac{1}{\sqrt{1-4x^2}} dx, \int x\sqrt{x-1} dx, \int \sqrt{x^2+2x+2} dx.$$

Exercise 4 – Use the integration by part and/or the substitution to find the primitive

$$\int x^2 e^x dx, \int \arccos(x) dx, \int \ln\left(1+\sqrt{x}\right) dx.$$

Exercise 5 – Find the primitive of the fractional functions

$$f_1(x) = \frac{x^3 - 4x^2 + 2x + 6}{x^2 - 5x + 6}, f_2(x) = \frac{1}{(x-1)(x^2-1)}$$
$$f_3(x) = \frac{1}{(x+2)(2x^2+1)}, f_4(x) = \frac{1}{x(+1)^2(x^2+2x+2)}.$$
$$f_5(x) = \frac{1}{(x^2+1)^2}$$

Exercise 6 – Use the appropriate method to find the primitive of the following functions

$$f(x) = \frac{\sin(2x)}{\sin^2(x) - 5\sin(x) + 6}, g(x) = \frac{\sinh^3(x)}{\cosh(x)}.$$

A supplementary exercise

Exercise 7 – Find the primitive

$$\int \frac{1}{(\sinh(x)+\cosh(x))^n} dx, (n \in \mathbb{N}), \int \frac{1}{x}\sqrt{\frac{x-1}{x+1}} dx, \int \frac{1}{\sqrt{x^2+1}} dx, \int \frac{1}{x^2+2x+5} dx,$$
$$\int \frac{x}{\sqrt{9+4x^4}} dx, \int \frac{\cos(x)\sin(x)}{\cos^4(x)+\sin^4(x)} dx, \int \frac{1}{(x+1)(x^2+2x+2)} dx, \int \frac{x^3}{x^2-2x+1} dx,$$
$$\int \frac{\cos(x)+1}{\sin(x)-1} dx, \int x\sqrt{-x^2+3x-2} dx, \int \frac{x}{(x+1)^3(x^2+1)} dx, \int \frac{\sin(x)}{\sin(x)+\cos(x)} dx,$$
$$\int \frac{1}{\cosh^3(x)+\sinh^3(x)} dx, \int \frac{x^4(x-1)^4}{x^2+1} dx, \int \frac{e^x}{e^{2x}+3e^x+2} dx, \int \frac{x^4+81}{x(x^2+9)^2} dx.$$