BADJI MOKHTAR ANNABA UNIVERSITY
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS 1st year /2024–2025
Transversal Teaching Unit
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Evaluation Mode: EMD Rating 60% and TD Rating 40%

I-Electrostatics

I.1 Coulomb's law

I.1.a. Definition of electric charge

The notion of electric charge is linked to the phenomenon of electrification. A body is said to be electrified if it has the property of attracting light bodies regardless of their nature. Experience shows that there are two types of charges, a positive (+) and a negative (-). Charges with the same signs repel each other and those with opposite charges attract each other.

According to modern physics, the carriers of elementary negative charges are electrons (e-) denoted e (e = -1.602 189 2 10^{-19} C), its mass is m_e = 0.910 954 4 10^{-30} Kg. The proton has a positive charge (q_p = 1.602 189 2 10^{-19} C), its mass is mp = 1.672 648 5 10^{-27}Kg. The mass of a neutron is m_n = 1.674 954 3 10^{-27}Kg.

An atom that loses an electron will be a positive ion, and An atom that gains an electron will be a negative ion. The unit of electric charge is the Coulomb [C]. According to the formula for electric current: I = q/t , q = It [q] = [I][t]=1A.1s=1 C

I.1.b. Statement of Coulomb's law

Two static point electric charges q_1 and q_2 exert on each other a force $\vec{F}$ directly proportional to the two electric charges and inversely proportional to the square of their distance r:

$$\vec{F} = K\frac{q_1 q_2}{r^2}\vec{u}$$

Such that k is a constant of proportionality $K = \frac{1}{4\pi\varepsilon_0}$, où ε_0= 8,854 187 82 10^{-12} Fm^{-1} is the permittivity in a vacuum.

From where K is almost equal to 910^9 we can write the force in this form: $\vec{F} = 910^9\frac{q_1 q_2}{r^2}\vec{u}$

The unit of force is the Newton; [F]= $\frac{[c][c]}{[m^2]} = [N]$

If q_1 and q_2 are >0 or q_1 and q_2 are <0, (F>0), then we have a repulsive force (figure 1)

If q_1>0 and q_2<0, (F<0), then we have an attractive force (figure 2)

As the force is a vector we are obliged to draw a diagram.

Example: At the tops of the angles of a right triangle A, B, C, where the distance r_1 = AC=1.2 m and r_2 = CB= 0.5 m, we place three charges $q_1 = 1.510^{-3}$C, $q_2 = -0.510^{-3}$C and $q_3 = 0.210^{-3}$C. (figure3).

Calculate the resulting electric force acting on the charge q_3.

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2$$

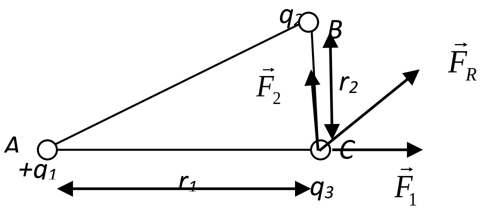


Figure3

$$F_R = \sqrt{F_1^2 + F_2^2}$$

$$F_1 = K\frac{q_1 q_3}{r^2} = 910^9 \frac{1{,}5.10^{-3}.0{,}210^{-3}}{(1{,}2)^2} = 1{,}875N$$

$$F_2 = K\frac{q_2 q_3}{b^2} = 910^9 \frac{0{,}2.10^{-3}.0{,}510^{-3}}{(0{,}5)^2} = 3{,}6N$$

$$F_R = \sqrt{(1{,}875)^2 + (3{,}6)^2} = 4{,}0610^{-3}N$$

I.2. The electric field E

Definition: Suppose that the charge q_1 is small enough not to significantly disturb the electric field by the charge q. We then define the electric field created by the charge q, as the force exerted on the unit of charge q_1. The electric field is a vector

$$\vec{E} = \frac{\vec{F}}{q_1} = \frac{Kq}{r^2}\vec{u} = \frac{q}{4\pi\varepsilon_0 r^2}\vec{u}$$

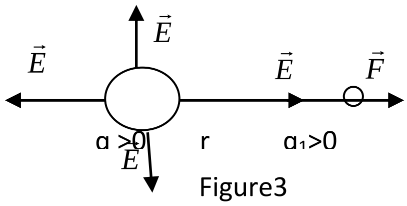


Figure3

The unit of electric field is volt per meter; $[E] = \frac{[V]}{[m]}$

(Like the Joule J= N m = e V $\Rightarrow N = \frac{CV}{m}$ $Comme [E] = \frac{[F]}{[q]} = \frac{N}{C} = \frac{CV}{mC} = \frac{V}{m}$)

I.3. The electric potential V

The electric potential is a scalar, which is directly linked to electrostatic energy.

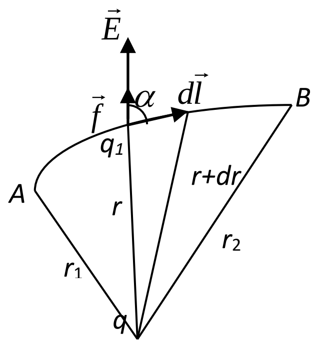
$$V = \frac{Kq}{r} = \frac{q}{4\pi\varepsilon_0 r}$$

The unit of electric potential is the volt [V]= [V]

Electrostatic Energy

a- Let a point charge q1 > 0 be sufficiently small, located in an electric field created by the charge q > 0, which is subject to the action of a force $\vec{f}$.

b- Let a displacement element, $d\vec{l}$ small so that the force $\vec{f}$ is constant in module and direction.



Elementary work $dW = \vec{f}.d\vec{l} = fdl\cos\alpha$

$$W = \int_A^B \vec{f}d\vec{l} = \int_A^B q_1\vec{E}d\vec{l} = \int_A^B q_1 Edl\cos\alpha = q_1\int_A^B Edl\cos\alpha$$

We call $W = \int_A^B \vec{E}d\vec{l}$ the circulation of the vector $\vec{E}$ between A and B

Knowing that the electric field E, created by the electric charge q, is:

$$E = \frac{q}{4\pi\varepsilon_0 r^2} \Rightarrow W = q_1\int_{r_1}^{r_2}\frac{q}{4\pi\varepsilon_0 r^2}dl\cos\alpha \Rightarrow W = \frac{q_1 q}{4\pi\varepsilon_0}\int_{r_1}^{r2}\frac{dr}{r^2} = \frac{q_1 q}{4\pi\varepsilon_0}[\frac{1}{r_1} - \frac{1}{r_2}]$$

The electric potential V

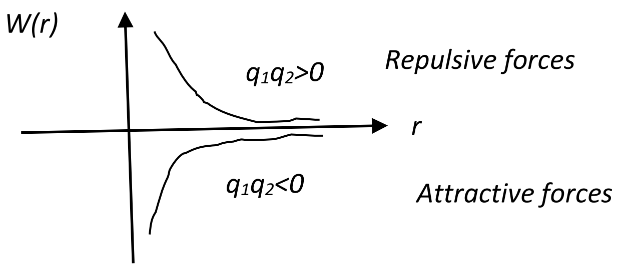
By definition the work is equal to the reduction in electrostatic energy, that is to say dW = -dEp, which can also be written dEp = - dW,

$$\Rightarrow dE_p = -\frac{q_1 q}{4\pi\varepsilon_0}\int\frac{dr}{r^2} \Rightarrow E_p = \frac{q_1 q}{4\pi\varepsilon_0 r} + C$$

$$When r \to \infty \Rightarrow E_p \to 0 \Rightarrow C = 0 \Rightarrow E_p = \frac{q_1 q}{4\pi\varepsilon_0 r}$$

We define the electric potential V as follows: $V = \frac{E_p}{q_1} = \frac{q}{4\pi\varepsilon_0 r}$, is the electric potential V created by the electric charge q. The unit of electric potential is the Volt ([V] $= \frac{[E_p]}{[q]} = \frac{J}{C} = V$)

$E_P = q_1V$, the unit of energy is the Joule (1eV = 1.6 10^{-19} J)



I.4. Principle of superposition

Distribution of point electrical charges

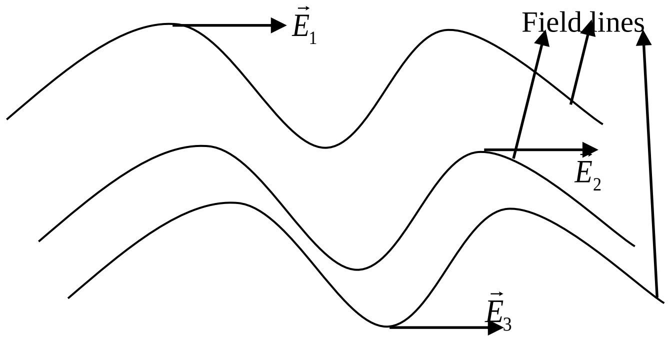
If we have $q_1, q_2, q_3, \ldots, q_n$

The resulting electric force $\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + .. + \vec{F}_n \Rightarrow \vec{F}_R = \sum_{i=1}^{n} \vec{F}_i$

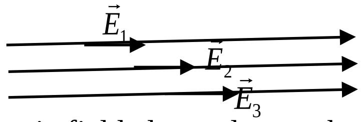
The resulting electric field $\vec{E}_R = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + .. + \vec{E}_n \Rightarrow E_R = \sum_{i=1}^{n} \vec{E}_i$

The resulting electric potential $V_R = V_1 + V_2 + V_3 + .. + V_n = \Rightarrow V_R = \sum_{i=1}^{n} V_i$

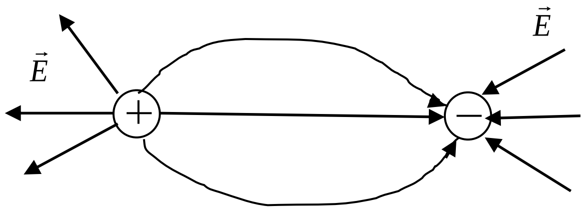
I.5 Electric field topography $\vec{E}$

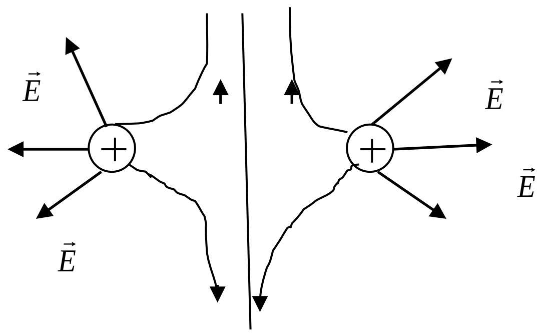


In the case of a uniform field



The electric field always leaves the positive (+) electric charge and enters the negative (-) electric charge.



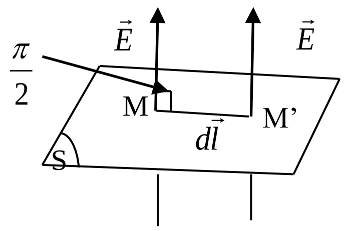


I.6. Equipotential surfaces

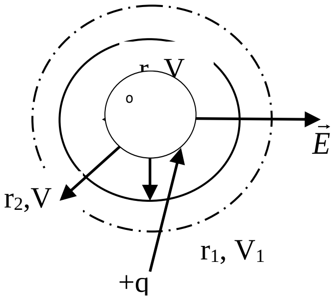
The electric field E is perpendicular to the surface S

Ep= q(V_M - $V_{M'}$) =0 si dEp=0 $\Rightarrow dE_p = dW = qEdl\cos\alpha = 0 \Rightarrow$

$q \neq 0, E \neq 0 et dl \neq 0 \Rightarrow \cos\alpha = 0 \Rightarrow \alpha = \frac{\pi}{2}$



Examples :a-Let us a sphere with center 0 and radius r charged on the surface by an electric charge +q. the potential electric V at the surface of the sphere is $V=\frac{q}{4\pi\varepsilon_0 r}$



It is equal to a constant, because the radius r constant. the electric potential V_1,

on the surface of the sphere of radius r_1, is $V_1=\frac{q}{4\pi\varepsilon_0 r_1}$

is equal to a constant, so the radius r_1 is equal to a constant. the electric potential V_2, on the surface of the sphere, is $V_2=\frac{q}{4\pi\varepsilon_0 r_2}$ is equal to a constant, so the radius r_2 is equal to a constant. So the equipotential surfaces are concentric spheres.

b-If the electric fields are uniform and continuous, the equipotential surfaces are parallel planes perpendicular to the electric fields.

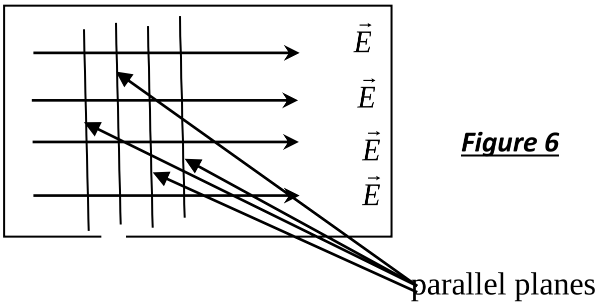
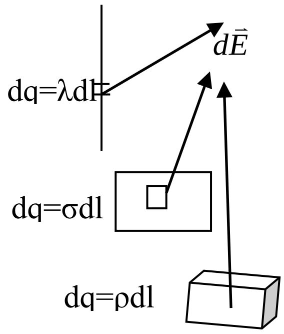


Figure 6

1.7.Distribution of electric charges



Distribution of linear electric charges

Let λ be the linear electric charge density $\lambda=\frac{dq}{dl}\Rightarrow[\lambda]=\frac{C}{m}$

Distribution of surface electric charges

Let σ be the surface electric charge density $\sigma=\frac{dq}{dS}\Rightarrow[\sigma]=\frac{C}{m^2}$

$\rho=\frac{dq}{dV}\Rightarrow[\rho]=\frac{C}{m^3}$

Distribution of volumetric electric charges

Let ρ be the volumetric electric charge density

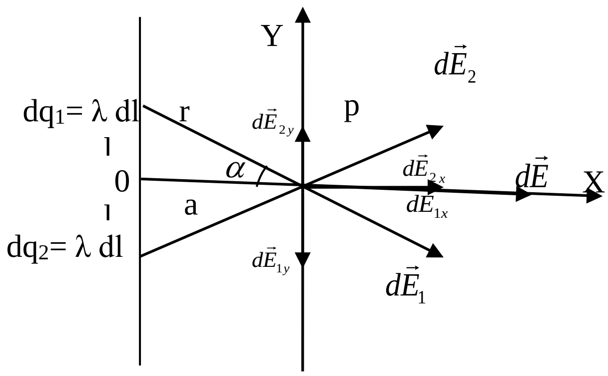
Application of the superposition principle for the calculation of an electric field

Consider a system of charges: a uniformly long wire, positively charged with a linear charge density λ. Find the electric field E, created by this distribution, at a distance a from the wire.

-Direct method

As $dq_1 = dq_2 = dq$

$\Rightarrow dE_1 = dE_2 = K\frac{dq}{r^2}=K\lambda\frac{dl}{r^2}$



According to the principle of superposition

$\vec{E}=\int d\vec{E}_i$

The projection of $d\vec{E}_1$ et $d\vec{E}_2$ along the axis P_Y

give dE_{1y} and dE_{2y} As $\left|\overrightarrow{dE_1}\right| = \left|\overrightarrow{dE_2}\right|$ and their directions are opposite which gives the resultant is zero on the Y axis. On the other hand, the projection along the P_x axis exists $dE = dE_{1x} = dE_{2x} = dE_1 \cos\alpha + dE_2 \cos\alpha = 2\, dE_1 \cos\alpha \Rightarrow E = 2K \int \frac{\lambda dl}{r^2} \cos\alpha$

We notice that dE is perpendicular to the wire due to the symmetry and as the wire is infinitely long. The law of superposition gives after a change of variable dl to dα, therefore the sum of E_i perpendicular to the wire can be swept between 0 and π/2.
According to the figure we draw:

$$tg\alpha = \frac{l}{r} \Rightarrow l = atg\alpha \Rightarrow dl = a\frac{d\alpha}{\cos^2\alpha} \text{ and as } \cos\alpha = \frac{a}{r} \Rightarrow r = \frac{a}{\cos\alpha} \Rightarrow r^2 = \frac{a^2}{\cos^2\alpha}$$

So we have: E= $E = 2K\lambda \int_0^{\pi/2} \frac{a\cos\alpha\, d\alpha}{\cos^2\alpha \frac{a^2}{\cos^2}} = \frac{2K\lambda}{a}\int_0^{\pi/2}\cos\alpha d\alpha = \frac{2K\lambda}{a}\left[\sin 1 - \sin 0\right]$

$$E = \frac{\lambda}{2\pi\varepsilon_0 a}$$

I.8. Electric dipole

A dipole is composed of two equal electric charges of opposite sign, separated by a small distance a. We call dipole moment $\vec{\mu} = q\vec{a}$, which is always directed from the charge –q to the charge +q.
The electric potential at point M is

$$V = V_A + V_B \quad \Rightarrow V_M = \frac{q}{4\pi\varepsilon_0 r_1} - \frac{q}{4\pi\varepsilon_0 r_2} \quad \Rightarrow V_M = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_2 - r_1}{r_2 r_1}\right)$$

As a << r $\Rightarrow$ $r_1 = r_2 = r$ et $r_2 - r_1 = a\cos\theta$

$V_M = \frac{qa\cos\theta}{4\pi\epsilon_0 r^2}$, knowing that the dipole moment $\vec{\mu} = qA\vec{B}$, $V_M = \frac{K\mu\cos\theta}{r^2} = \frac{\mu\cos\theta}{4\pi\varepsilon_0}$

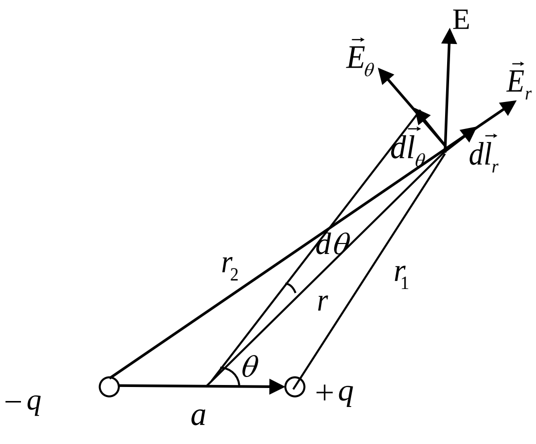
The unit of dipole moment [μ]= [C][m] (1Debye [D]= $\frac{1}{3}10^{-29} \approx 3{,}3310^{-30} Cm$.

Electric Field
Components E_r et E_θ
According $\vec{E} = -grad\vec{V}$ we can write

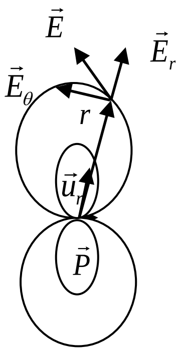
$$dV = -\vec{E}d\vec{l} \Rightarrow \frac{\partial V}{\partial r}dr + \frac{\partial V}{\partial \theta}d\theta = -(E_r dl_r + E_\theta dl_\theta)$$

$$\Rightarrow \frac{\partial V}{\partial r}dr + \frac{\partial V}{\partial \theta}d\theta = -(E_r dr + E_\theta r d\theta)$$

$$\Rightarrow E_r = -\frac{dV}{dr} \text{ et } E_\theta = -\frac{1}{r}\frac{dV}{d\theta}$$



$E_r = \dfrac{p\cos\theta}{2\pi\varepsilon_0 r^3}$ et $E_\theta = \dfrac{p\sin\theta}{4\pi\varepsilon_0 r^3}$



Calculation of the modulus of E

$$E = \sqrt{E_r^2 + E_\theta^2} \quad \Rightarrow E = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{1+3\cos^2\theta}$$

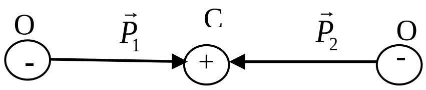
$$Si.\theta = 0 \Rightarrow E = \frac{2KP}{r^3} = E_r \Rightarrow E_\theta = 0 \Rightarrow V = \frac{KP}{r^2}$$

$$Si.\theta = \frac{\pi}{2} \Rightarrow E = \frac{KP}{r^3} = E_\theta \Rightarrow E_r = 0 \Rightarrow V = 0$$

Examples :

1- Case of CO_2

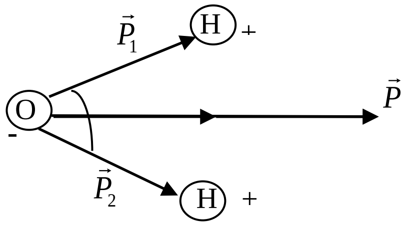
$$\Rightarrow \vec{P} = \vec{P}_1 + \vec{P}_2 = 0$$



The dipole moment of CO_2 est nul.

2- Case of H_2O

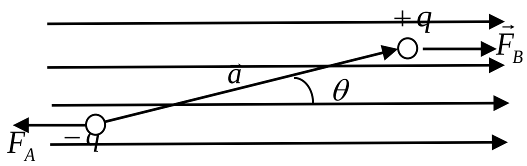
$$\Rightarrow \vec{P} = \vec{P}_1 + \vec{P}_2 \Rightarrow P = P_1\cos\frac{105}{2} + P_2\cos\frac{105}{2} = 2P_1\cos\frac{105}{2}$$



Electric dipole in a uniform electric field

we have $\vec{F}_A = -q\vec{E}_0$ et $\vec{F}_B + q\vec{E}_0$ the resulting force

$$\vec{F} = \vec{F}_A + \vec{F}_B \Rightarrow F = F_A - F_B = 0$$



The forces $\vec{F}_A$ et $\vec{F}_B$ cause a moment force couple

$$M = qE_0 a\sin\theta = PE_0\sin\theta \Rightarrow \vec{M} = \vec{P}\wedge\vec{E}_0$$

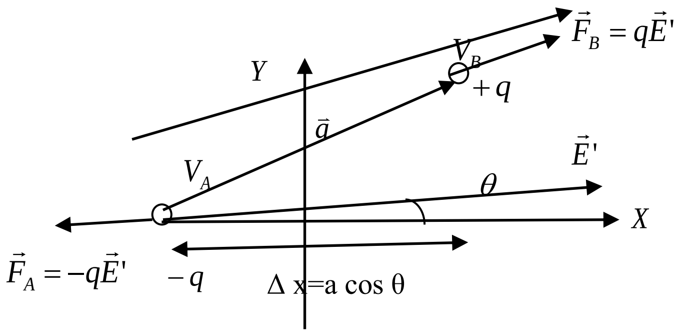
This moment tends to rotate the electric dipole following the direction of the electric field lines.

Electric dipole in a non-uniform electric field

On a $dW = -dE_P$ le travail élémentaire

$$\int_A^B \vec{F}d\vec{l} = -q\int_A^B dV = \int_A^B q\vec{E}d\vec{l} = -q(V_B - V_A)$$

$$\vec{E}.\vec{AB} = -(\Delta V)$$



$E_P = E_{PA} - E_{PB} = q\,(\Delta V) = -q\vec{E}.\vec{AB}$

The forces $\vec{F}_A$ et $\vec{F}_B$ cause a moment force couple

$\Rightarrow E_P = -\vec{P}.\vec{E} = PE\cos\theta$

If θ = 0 $\Rightarrow E_P = -PE$ (mini) at the equilibrium position, the dipole aligns with field E

If θ = π/2 $\Rightarrow E_P = PE$ (maxi)

1.9.Gauss's theorem

Flow of an electric field through a closed surface.

$d\phi = \vec{E}.d\vec{S} \Rightarrow \phi = \oint_S \vec{E}.d\vec{S}$ Where $d\vec{S}$ is directed from the inside to the outside.

Flow of an electric field $\vec{E}$ through a closed surface (S) is equal to the sum of the electric charges found inside this closed surface (S) divided by ε_0

$$\phi = \oint_S \vec{E}.d\vec{S} = \sum_{i=1}^{n} \frac{q_i}{\varepsilon_0} = \int_{i=1}^{n} \frac{dq_i}{\varepsilon_0}$$

Conditions for applying Gauss' theorem:

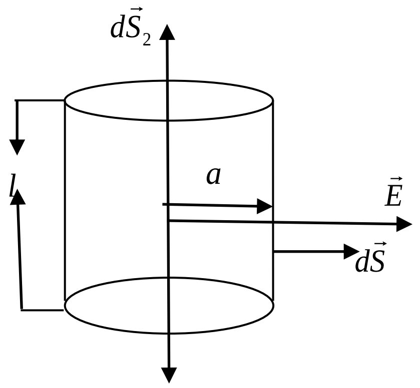
a-It is necessary to choose a surface containing semmetry the electric field $\vec{E}$ is constant on this surface.

b-We must choose a closed surface that is easy to calculate.

Calculation of an electric field E

1-Case of a uniformly long, positively charged wire with a linear charge density λ. Find the electric field E, created by this distribution, at a distance a from the wire.

- Gauss' theorem: $\Phi = \oint_S \vec{E}d\vec{S} = \int \frac{dq}{\varepsilon_0}$



$\Phi = \oint_S \vec{E}d\vec{S} + \oint_S \vec{E}d\vec{S}_1 + \oint_S \vec{E}d\vec{S}_2 = \int \frac{dq}{\varepsilon_0}$

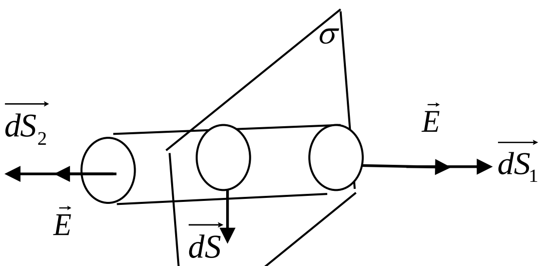
$\Phi = \oint_S \vec{E}d\vec{S} = \int \frac{dq}{\varepsilon_0} = \oint_S \vec{E}d\vec{S} = \oint_S EdS\cos\theta = \int \frac{dq}{\varepsilon_0}$

$\Phi = E\oint_S dS = \int \frac{dq}{\varepsilon_0} = \int_0^l \frac{\lambda dl}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$

E. $S_3 = \frac{\lambda l}{\varepsilon_0} \Rightarrow E.2\pi a l = \frac{\lambda l}{\varepsilon_0} \Rightarrow \frac{\lambda}{2\pi\varepsilon_0 a}$

2- Case of a uniformly infinite plane, positively charged with a surface charge density σ. Find the electric field E, created by this distribution, near the plane.

Gauss' theorem: $\Phi = \oint_S \vec{E}d\vec{S} = \int \frac{dq}{\varepsilon_0}$



$$\phi = \oint_S \vec{E}d\vec{S} + \oint_S \vec{E}d\vec{S}_1 + \oint_S \vec{E}d\vec{S}_2 = \int \frac{dq}{\varepsilon_0}$$

As E is perpendicular to dS

$$\phi = \int EdS_1 \cos 0 + \int EdS_2 \cos 0 = \int \frac{dq}{\varepsilon_0}$$

As $dS_1 = dS_2 = dS$

$$\phi = 2\int EdS = \int \frac{dq}{\varepsilon_0} = \int_0^S \frac{\sigma dS}{\varepsilon_0} = \frac{\sigma S}{\varepsilon_0} \qquad 2E.S = \frac{\sigma S}{\varepsilon_0} \Rightarrow \frac{\sigma}{2\varepsilon_0}$$

Application exercise

We consider in a vacuum a charge distribution of volumetric and uniform charge density ρ in a spherical volume (S) of radius R.

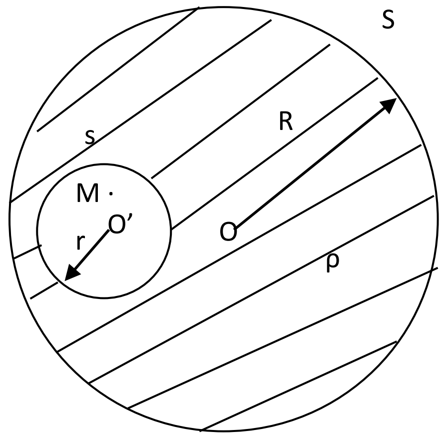
a) Calculate the electric field E at a point M at the distance OM < R from the center O.

b) The charges of the previous distribution are removed in a spherical space (s), with center O', with radius r, entirely inside (S).

Determine the electric field E at a point M in the volume(s) devoid of charges.

The system being of spherical symmetry, the electric field E is radial, depends only on the distance OM from the center, it is directed along OM.

Then by applying Gauss's Theorem to a spherical surface with center O, of radius OM = r, of surface $4\pi r^2$ which contains



The charge $\frac{4}{3}\pi r^3 \rho$, we obtain for the electric field E,

in size and direction $\vec{E} = \frac{\rho \vec{OM}}{3\varepsilon_0}$

b) The charge distribution described results from The addition to the previous charges of the charges Charge density – ρ produced in volume (s), To which the electric field they produce is added vectorially to the electric field which has just been determined, since the general expression of the field electrical is linear as a function of the acting charges. The charge density – ρ produced in the volume (s),

O'M < r, the electric field E 'given by the expression

Previous applied to the charge density – ρ and to the center O'.Let be: $\vec{E}' = -\frac{\rho \vec{O'M}}{3\varepsilon_0}$

The total electric field in space (s), produced by the distribution of charge density ρ in the volume (S) – (s), and of charge density ρ – ρ = 0 in (s), is therefore

$$\vec{E}_t = \vec{E} + \vec{E}' = \frac{\rho}{3\varepsilon_0}(\vec{OM} - \vec{O'M}),$$

Let be, $\vec{E}_t = \frac{\rho \vec{OO'}}{3\varepsilon_0}$,

Uniform, intensity $\frac{\rho OO''}{3\varepsilon_0}$ and direction OO'.