BADJI MOKHTAR ANNABA UNIVERSITY
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS 1st year /2024–2025
Transversal Teaching Unit
Responsible for the electricity module (Coefficient: 2, Credit: 3): Professor A. GASMI
Evaluation Mode: EMD Rating 50% and TD Rating 50%

Conductors in electrostatic equilibrium

A conductor is a body in which free charges can move freely. In the case of a metal or a semiconductor these are the free electrons. In an electrolyte these are the ions.

- Properties of a conductor in equilibrium

a- Inside the conductor the electric field is zero (E= 0)

b- The electric charge q is zero (q=0) inside the conductor

As it is a conductor, so the charges are near the surface

E=0, According to $\vec{E} = -grad\vec{V} \Rightarrow E = -\frac{dV}{dr} \Rightarrow V = cons\tan te$

This shows that the exterior surface is an equipotential surface; that is to say that the electric potential is the same on this surface. The electric field is always perpendicular to the latter.

We assume that the electric field in the vicinity of a conductor is:

$E = \frac{\sigma}{2\varepsilon_0\varepsilon_r}$ $ou.\sigma = \frac{q}{S}$ Is the electric charge density per unit area

B-Electrostatic pressure

$$p = \frac{F}{S} = \frac{qE}{S} = \sigma E = \frac{\sigma^2}{2\varepsilon_0}$$

C- Capacity of an insulated conductor

We define the capacity of a conductor as follows:C = q/V

The unit of capacitance is Farad [F]. We often use derived units which are:

1µF=10^{-6} F, 1nF = 10^{-9} F et 1pF = 10^{-12} F

D- Electrostatic energy of a conductor

$$W = \int V dq \int_0^Q \frac{q}{C} = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

E- Electrostatic influence phenomenon

When we place a conductor in an electric field E created by a charge q, it polarizes. In fact, it contains free electric charges which move in the opposite direction to the field. Positive and negative charges appear on either side of the conductor (figure 1)

Effect of electric field on a conductor

$$\vec{E}_{\text{int}} = \vec{E} + \vec{E}' = 0$$

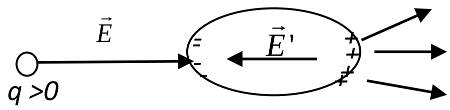


Figure 1

On dit que le conducteur est équilibre électrostatique et polarisé.

$$C = \left| \frac{q}{V_+ - V_-} \right| = \frac{q}{\Delta V}$$

The charge of a conductor has not changed but there has been a distribution of charges and also a variation in the electric potential.

<u>E-Corresponding Elements Theorem</u>

Given two conductors in equilibrium, a field tube cuts elements ds_1 and ds_2 on their surfaces; corresponding elements (figure 2).Calcul du flux à travers le tube de champ en appliquant

Gauss' Theorem.

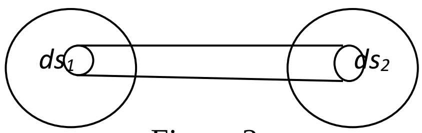


Figure 2

$$\oint_S \vec{E} d\vec{S} = \frac{dq_1 + dq_2}{\varepsilon_0} = 0$$

Because the field inside Eint= 0, the field E is parallel to the side surface of field tube. Thus the charge contained in the field tube is zero.

$\sigma ds_1 = \sigma ds_2$ This constitutes the corresponding elements Theorem.

E1-Partial influence

We note that the lines of the field coming from the driver A do not all arrive on the driver B.
According to the Corresponding Elements Theorem

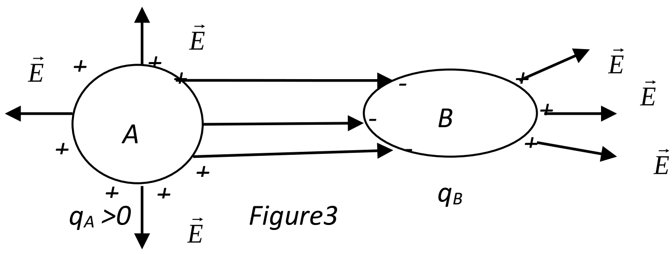


Figure3

$$|q_A| > |q_B|$$

<u>E2-Total influence</u>

Conductor C_2 completely surrounds the influencing conductor C_1.

1st case: C_2 originally neutral, therefore inside C_2 we have

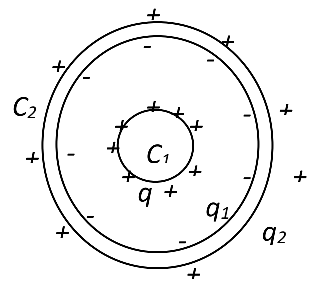


Figure 4

E= 0 so $q_1 = -q_2$. If C_2 is isolated and neutral, we therefore have

$q_1 = -q$ et $q_2 = +q$

2nd case: C_2 is linked to the ground

Under these conditions the potential is zero and consequently

The charge $q_2 = 0$ and only $q_1 = -q$ remains.

F- Dielectric

Is an insulator that stores energy which becomes available as soon as the field is removed.

It increases the capacity of capacitors.

Table 1: The permittivities of some media

Dielectric	Air	Paraffin	Paper	Mica	Porcelain	Barium Titanium
ε_r	1,000	2,1	2 à 6	5 à 8	5 à 7	1750

G- Capacitors

1. Plane capacitor:

Consists of two parallel plane armatures which are separated by a small distance d, filled with a dielectric of permittivity ϵr, and thickness d.

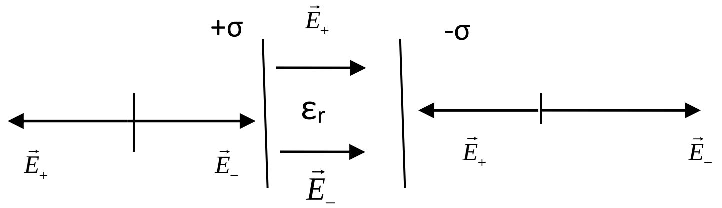


Figure 5

$$\vec{E} = \vec{E}_+ + \vec{E}_- \Rightarrow E = E_+ + E_- = \frac{\sigma'}{2\varepsilon_0\varepsilon_r} + \frac{\sigma}{2\varepsilon_0\varepsilon_r} = \frac{\sigma}{\varepsilon_0\varepsilon_r}$$

As $$\sigma = \frac{q}{S} \Rightarrow E = \frac{q}{S\varepsilon_0\varepsilon_r}$$

As $$V = V_1 - V_2 = \int_0^d E dl = Ed \Rightarrow V = \frac{qd}{S\varepsilon_0\varepsilon_r}$$

$$C = \frac{q}{V} \; donc \; \frac{q}{V} = \frac{d}{S\varepsilon_0\varepsilon_r} \Rightarrow \frac{1}{C} = \frac{d}{S\varepsilon_0\varepsilon_r} \Rightarrow C = \frac{\varepsilon_0\varepsilon_r S}{d}$$

2. Spherical capacitor

Consider a spherical layer, inside the sphere, of radius r and thickness dr and surface s= 4πr2 therefore of elementary volume dv = 4πr2 dr (figure 6).

Like elementary electrostatic energy $dE_P = \frac{\varepsilon_0\varepsilon_r E^2}{2} dV$

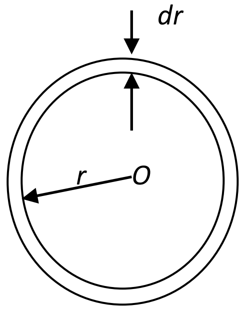


Figure 6

As $4\pi\, r^2\, dr$

So we have $dE_P = \frac{\varepsilon_0\varepsilon_r}{2}\left(\frac{q^2}{16\pi^2\varepsilon_0^2\varepsilon_r^2 r^4}\right)4\pi r^2 dr$

$$dE_P = \frac{q^2 dr}{2.4\pi\varepsilon_0\varepsilon_r r^2}$$

$$E_P = \int_{R1}^{R2} \frac{q^2}{2.4\pi\varepsilon_0\varepsilon_r}\frac{dr}{r^2} = \; = \frac{q^2}{2.4\pi\varepsilon_0\varepsilon_r}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{q^2}{2C} \Rightarrow C = \frac{4\pi\varepsilon_0\varepsilon_r R_1 R_2}{R_2 - R_1}$$

. cylindrical capacitor

We consider a cylindrical capacitor, of length l and radii R_1 and R_2 and of charge –q and +q respectively (figure 7).

a) Knowing that $R_1 < R_2$, and that the dielectric constant of the medium is εr and that l >> in front of R_1 and R_2.

b) - Gauss's theorem: $\Phi = \oint_S \vec{E}d\vec{S} = \int \frac{dq}{\varepsilon_0\varepsilon_r}$

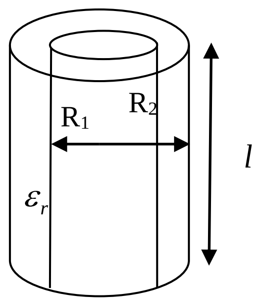


Figure 7

For r < R_1 et r > R_2 on a $\Phi = \oint_S \vec{E}d\vec{S} = \sum \frac{q_i}{\varepsilon_0\varepsilon_r} = 0$ ⇒ E=0

For R_1< r < R_2 on a $\Phi = \oint_S \vec{E}d\vec{S} = \sum \frac{q_i}{\varepsilon_0\varepsilon_r}$

E. S = $\frac{q}{\varepsilon_0\varepsilon_r} \Rightarrow E.2\pi rl = \frac{q}{\varepsilon_0\varepsilon_r} \Rightarrow E = \frac{q}{2\pi\varepsilon_0\varepsilon_r l}$

c) Knowing that electrostatic energy is $dE_P = \frac{\varepsilon_0\varepsilon_r E^2}{2} dV$

As dV=2πrdr

So we have $dE_P = \frac{\varepsilon_0\varepsilon_r}{2}\left(\frac{q^2}{4\pi^2\varepsilon_0^2\varepsilon_r^2 r^2 l^2}\right) l 2\pi r dr$

$$dE_P = \frac{q^2 dr}{2.2\pi\varepsilon_0\varepsilon_r rl}$$

$$E_P = \int_{R1}^{R2} \frac{q^2}{2.2\pi\varepsilon_0\varepsilon_r l}\frac{dr}{r} = = \frac{q^2}{2\pi\varepsilon_0\varepsilon_r l}\log\frac{R_2}{R_1}$$

$$E_P = \frac{q^2}{2\pi\varepsilon_0\varepsilon_r l}\text{Log}_{-}\frac{R_1}{R_2} = \frac{q^2}{2C} \Rightarrow \text{C} = \frac{2\pi\varepsilon_0\varepsilon_r l}{\log\frac{R_2}{R_1}}$$

F. Association of capacitors

In parallel

$$Ceq = \sum_{i=1}^{n} C_i$$

Serial

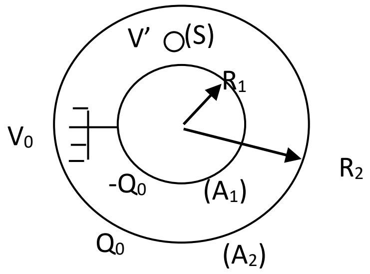
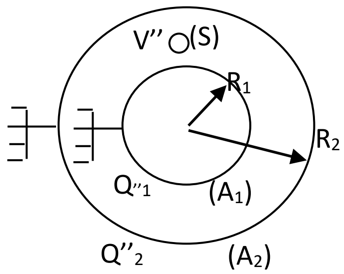
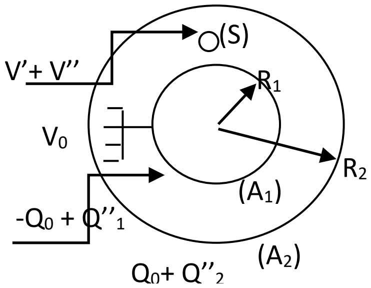
$$\frac{1}{Ceq} = \sum_{i=1}^{n}\frac{1}{C_i}$$

Exercice d'apllication

Entre les armatures (A_1) et (A_2) d'un condensateur sphérique vide, de rayon R_1 et R_2, maintenues aux potentiels 0 et V_0, on dispose d'une charge ponctuelle q à la distance r du centre.
Calculer les charges sur les armatures en supposant par commodité que q est porté par un conducteur (S) de dimensions négligeables.

On peut considérer l'état d'équilibre su système comme le résultat de la superposition de deux états d'équilibre dont le premier correspond au potentiel V_0 appliqué à (A_2), (A_1) étant le sol pour fixer l'origine, avec une charge nulle sur (S) dont le potentiel est alors V' et le second à potentiel nul pour (A_1) et (A_2) et la chrge q sur (S) dont le potentiel est alors V''.

Tableau 2 : caractérisent ces résultats de même que leur superposition

Figures	Potentiels			Charges		
	(A_1)	(A_2)	(S)	(A_1)	(A_2)	(S)
Etat 1 	0	V_0	V'	$Q'_1 = -Q_0$	$Q'_2 + Q_0$	0
Etat 2 	0	0	V''	Q''_1	Q''_2	q
Etat résultant 	0	V_0	V'+ V''	$-Q_0 + Q''_1$	$Q_0 + Q''_2$	q

Les charges $+Q_0$ et - Q_0 de l'état1 sont celles du condensateur sphérique sous la tension V_0 soit

$$Q_0 = \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1} V_0 \qquad (1)$$

Les charges Q''$_1$ et Q''$_2$ de l'état 2 sont développées par influence sous l'effet de q, donc telles que

$$Q''_1 + Q''_2 = -q \qquad (2)$$

Par ailleurs, ces états d'équilibre obeissent à l'identité de Gauss qui donne

$$Q''_2 \; V_0 + q\, V' = 0. \qquad (3)$$

Mais le potentiel V' dans l'état1du condensateur infiniment petit (S) qui ne perturbe pas la distribution entre (A_1) et (A_2) est

$$V' = \frac{Q_0}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{r}\right) \qquad (4)$$

On en déduit alors de (3) compte tenu de (4) et (1),

$$Q''_2 = \frac{R_2(r - R_1)}{(R_2 - R_1)r} q$$

Puis, avec (2),

$$Q''_1 = \frac{R_1(R_2 - r)}{(R_2 - R_1)r} q$$

Les charges -Q_0 et + Q''$_1$ et Les charges +Q_0 et +Q''$_2$ portées par les armatures dans l'état résultant sont ainsi

$$Q_0 + Q''_1 = \frac{-R_1}{(R_2 - R_1)}\left(4\pi\varepsilon_0 R_2 V_0 + \frac{R_2 - r}{r} q\right)$$

Et

$$Q_0 + Q''_2 = \frac{R_2}{(R_2 - R_1)}\left(4\pi\varepsilon_0 R_1 V_0 + \frac{r - R_1}{r} q\right)$$