BADJI MOKHTAR ANNABA UNIVERSITY
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS 1st year /2024–2025
Transversal Teaching Unit
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Evaluation Mode: EMD Rating 50% and TD Rating 50%

Electrokinetics

1-electric current

We call the electric current I, the ordered movement of electric charges.

By convention we take the direction of the electric current as the direction of movement of the positive electric charges (+)

(+) ⟶ movement of electric charges $\Rightarrow$ direction of electric current

In metals, electric current is induced by the movement of free electrons (e-). We define the electric current I by:

$I=\frac{dq}{dt}$, its unit is the Ampere $[I] = \frac{[dq]}{[dt]} = \frac{C}{s} = A$

2- Current density

Consider a homogeneous conductor AB of cylindrical shape with section S traversed by an electric current I. The latter is induced by the action of the electric field $\vec{E}$ (directed according to the minimum of the potential).

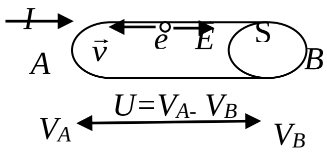


Figure 1

$(V_A > V_B)$

Under the action of an electric field $\vec{E}$, the electrons (e-) accelerate and then collide with the ions which constitute the crystal lattice. After the shock the (e-) accelerate again. As a result, the average speed of (e-) is very small (of the order of 2mm/s), because the real speed between two shocks is of the order 10^5m/s. Let be n, the number of charged particles per unit volume, and e is the charge of the electron (e-). The elementary electric charge which travels through the section S, during the time dt is: dq = neSvdt ; as l = v dt $\Rightarrow$ dq = neSl ; as the volume V= Sl $\Rightarrow$ dq= neV = Ne

$I = \frac{dq}{dt} = neSv$. We know that the electric current density $J = \frac{I}{S} = nev$

$$\vec{J} = \frac{I}{S} = ne\vec{v}$$

3- in differential form

Under the action of the electric field $\vec{E}$ the electron (e-), of mass m, is subjected to an electric force F_e. At equilibrium the mechanical force F_m is equal to an electric force F_e

$F_m = F_e$ which is also written, m a = eE, $\Rightarrow$ the acceleration $a = \frac{e}{m}E = \frac{dv}{dt} = \frac{e}{m}E$

$$\Rightarrow dv = \frac{e}{m}Edt \Rightarrow \int_0^{V_{Max}} dv = \int_0^{\tau} \frac{e}{m}Edt \Rightarrow V_{Max} = \frac{e}{m}E\tau$$

average speed $v_{Moy} = \frac{V_{Max} - V_{Min}}{2} \Rightarrow v_{Moy} = \frac{e}{2m}E\tau$

current density $J = nev = ne\frac{e}{2m}E\tau = \frac{ne^2}{2m}\tau E \;\; J = \gamma E$

Ohm's law in differential form is therefore written $J = \gamma E$ such as electrical conductivity

$$\gamma = \frac{ne^2}{2m}\tau$$

$$\vec{J} = \gamma\vec{E}$$

We call the resistivity, unit Ohm meter [ρ]= Ωm, $\rho = \frac{1}{\gamma}$

As the conductor is homogeneous of length l with section S. As the reduction in potential, from A to B, is linear, this gives, the electric field E:

$E = \frac{V_A - V_B}{l} \Rightarrow V_A - V_B = El$. According to Ohm's law in differential form we draw $E = \frac{J}{\gamma}$

So $V_A - V_B = \frac{J}{\gamma}l$ .As $J = \frac{I}{S}$ So we have $V_A - V_B = \frac{I}{S\gamma}l$ As $\rho = \frac{1}{\gamma}$ So we have

$V_A - V_B = \frac{I\rho l}{S}$. We pose $R = \frac{\rho l}{S}$ which represents resistance of unit Ohm [R] = Ω

$V_A - V_B = RI$, It is Ohm's law.

4- joule law

The electron (e-) gives energy to the atom during the shock in the form of kinetic energy $E_C = \frac{1}{2}mV_{Max}^2 = \frac{m}{2}(\frac{eE\tau}{m})^2$, the electron (e-) undergoes $\frac{1}{\tau}$ shock gives the atoms energy

$$\frac{W_1}{\tau} = \frac{e^2E^2}{2m}\tau$$

We call the energy given by the (e-) per unit of time and per unit of volume:

$W_0 = n\frac{W_1}{\tau} = \frac{ne^2\tau}{2m}E^2 = \gamma E^2$ As J= γ E $\Rightarrow W_0 = JE$. Joule's law in differential form is therefore written as the displacement of the charge dq gives work dA = Udq = UIdt

$$A = \int_0^t UIdt = UIt \text{ As } V_A - V_B = RI = U \Rightarrow A = RIIt = RI^2t$$

$A = W = Q = RI^2t$, is Joule's law (it is a quantity of heat; it is work dissipated in the form of heat).

Of unit the Joule [Q] =J

Power : $P = \frac{W}{t} = RI^2$

Resistance Association

In series, the equivalent resistance Re is:

$$R_e = \sum_{i=0}^{n} R_i$$

In parallel, the equivalent resistance Re is:

$$\frac{1}{R_e} = \sum_{i=1}^{n} \frac{1}{R_i}$$

Electric generator

Is a device that transforms mechanical, chemical, heat energy into electrical energy.

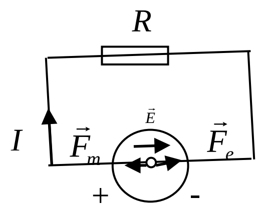
It is characterized by:

a- Electromotive force (e.m.f.) denoted E of unit Volt, [E]=V

b- Internal resistance r of unit Ohm, [r]=Ω

c- Its polarity plus (+) and minus (–)

the following electrical assembly:



$V_A > V_B$

Figure 2

For the electric current to be constant, $V_A - V_B$ must = Constant. This leads us to connect a device which transports the positive electrical charges from the potential V_A to V_B, this device nothing other than a generator.

Energy balance

The work supplied A is equal to the electrical work A' plus the work of the resisting forces A''

$A = A' + A'' = q(V_A - V_B) + qIr$

We call electromotive force (f. e.m.) noted E

$E = A/q = (V_A - V_B) + r\,I \Rightarrow E = V_A - V_B + Ir$

Across the resistance R, the electrical voltage is:

: $U = E - Ir$

Symbol of a generator

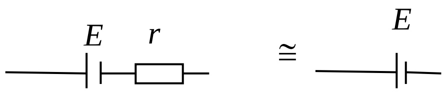


Figure 3

Electrical receiver

Is a device that transforms electrical energy W into another form of energy W' other than energy in the form of heat W''. It is characterized by:

a- Counter electromotive force (f.c.e.m), denoted E' of unit Volt, [E'] = V

b- Internal resistance r' of unit Ohm,[r']=Ω.

Energy balance

$W = W' + W'' = q(V_A - V_B) + W' + qIr'$

We call counter electromotive force (f. c. e.m.) noted E'

$E' = W'/q = (V_A - V_B) - r\,I \Rightarrow E' = (V_A - V_B) - Ir$

Symbol of a receiver

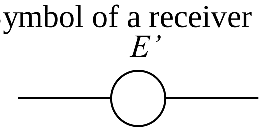


Figure 4

Kirchhoff's laws

Definitions:

1- A branch is made up of one or a set of electrical circuit elements, (resistance R, capacitance C, generator E, receiver E'…), associated in series between two nodes.
2- A node is a point common to at least three branches.
3- A mesh: is a set of electrical circuit elements associated in series thus forming a closed circuit.
4- An electrical network: is a set of electrical branches associated in any way.

1st law (law of knots)

The algebraic sum of the electric current intensities in a node is zero. We count the current leaving the node positively and the current entering it negatively.

$$\sum_{i=1}^{n} I_i = 0$$

2nd law (law of meshes)

$$\sum_{i=1}^{n} \pm E_i \pm \sum_{j=1}^{n} E' \pm \sum_{k=1}^{n} I_k R_k = 0$$

Such that E is the (f.e.m.), E' is the (f.c.e.m.), R is the resistance and I is the electric current.

In order to properly write this relationship we must:

Choose arbitrarily the direction of travel of the stitch.
b- We put the + sign in front of the Ei terms if we encounter the (+) pole of the generator and the – sign otherwise.
c- We precede the terms Ei' and $I_k R_k$ with a + sign if the direction of the electric current is that of the flow of the mesh and with a sign – otherwise.
Generally we know the Ei, Ei' and $I_k R_k$, we are asked to find the intensities of the electric current in all the branches. To find them we have a system of b equations with b unknowns, b being the number of branches.

Let M= $m_1 + m_2$ such that m_1 is the number of node equations and m_2 is the number of mesh equations.

We have m_1 = n-1, where n is the number of nodes

And m_2 = b- (n-1), where b is the number of branches

M = $m_1 + m_2$ = n-1+ b- (n-1) = b

Practical rule for solving an electrical network problem
1- Designate each intensity by a letter
2- Arbitrarily set the direction of the electric current
3- Write n-1 knot equations
4- Choose an arbitrary direction of the stitch's path (or mesh path).
5- Write b-(n-1) mesh equations
Note: if we find a negative current, it means that we have made an incorrect choice of the direction of the current.

Thévenin's theorem

Statement: In sinusoidal regime, any linear control dipole is equivalent, with respect to a load, to a single voltage generator whose d.d.p. is the no-load voltage of the previous dipole and whose internal impedance is the output impedance obtained by replacing all the voltage generators with short circuits and removing all the current generators.

Example: Consider a control dipole, with a potentiometric source $V= V_A - V_B$.

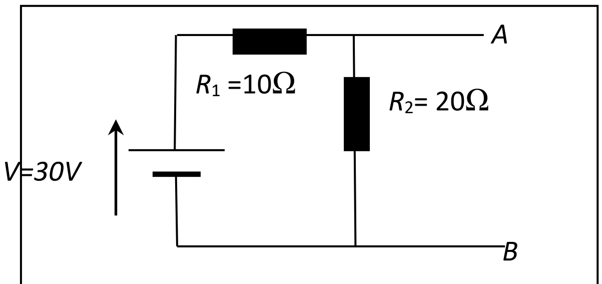


Figure 5

According to Ohm's law U=V= RI we draw the current $I = \frac{V}{R_e} = \frac{30}{20+10} = 1A$

When the dipole is empty, that is to say the open circuit voltage V_V, that is to say the voltage across the disconnected terminals of AB, we will have $V_V= V_{AB} = R_2\, I = 20.\ 1= 20V$

The current circuit electric current Icc, is the current which passes in a conductor without resistance connecting A and B. which means that the poles A and B have the same potential, therefore the resistance R2, shunted by the circuit course, is subject to a d.d.p. zero, it is therefore not crossed by a current. The electric current of the circuit $I_{cc} = \frac{V}{R_1} = \frac{30}{10} = 3A$

Replacing the potentiometric source with a short circuit allows us to calculate the impedance between A and B.

The equivalent resistance is $\frac{1}{R_e} = \frac{1}{10} + \frac{1}{20} = \frac{3}{30} \Rightarrow R_e = 6{,}66\Omega$ we check that it is equal to

$$R = \frac{V_V}{I_{cc}} = \frac{20}{3} = 6{,}66\Omega$$

We have an equivalent circuit (figure6)

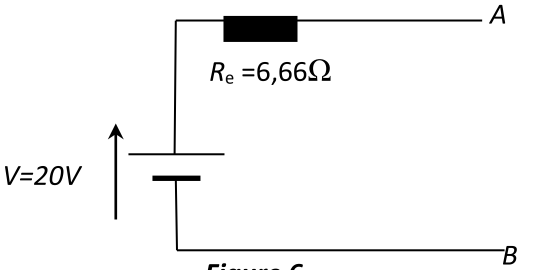


Figure 6

We can verify that the electric current which passes through a load connected between terminals A and B is the same when calculated with one or the other circuit, for example, let R= 20Ω the load.

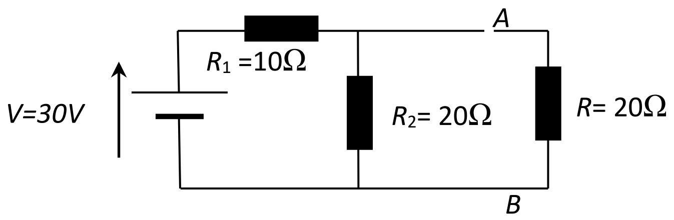


Figure 7

In order to calculate the current I generated by the generator, we replace the two resistances in parallel of 20 Ω with an equivalent resistance:

$$\frac{1}{R_e} = \frac{1}{20} + \frac{1}{20} = \frac{2}{20} \Rightarrow R_e = 10\Omega$$

So we have $I = \frac{V}{R_e} = \frac{30}{20} = 1{,}5A \Rightarrow V_A - V_B = V_{AB} = 10.1{,}5 = 15V$

The current I in the resistor R=20 Ω is: $I = \frac{V_{AB}}{R} = \frac{45}{20} = 0{,}75A$

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