1^{st} year Bachelor's degree in Mathematics
2024-2025, Semester 2 Series 3: The Riemann integral **Course: Analysis 2**

Exercise 1 – Let S be the step function defined in $[0, 4]$ by

$$S(x) = \begin{cases} -1, & \text{if } 0 \le x < 2 \\ 3, & \text{if } 2 \le x \le 4. \end{cases}$$

1. Write the partition of $[0, 4]$ and draw the curve of S.
2. Compute the Riemann integral of S over $[0, 4]$.
3. What does the Riemann integral of S over $[0, 4]$ represent ?

Exercise 2 – Let $f : [0, 3] \to \mathbb{R}$ be the function

$$f(x) = \begin{cases} 2, & \text{if } 0 \le x < 1 \\ x, & \text{if } 1 \le x < 2 \\ -4, & \text{if } 2 \le x \le 3. \end{cases}$$

1. Prove that f is Riemann integrable over $[0, 3]$ and use the definition to evaluate $\int_0^3 f(x)dx$.
2. Find the area between the curve $y = f(x)$ and the line $y = 1$

Exercise 3 – Use the fundamental theorem of calculus to evaluate the following

$$\int_e^{e^2} \frac{1}{x\ln(x)}dx, \int_0^4 \frac{1}{1+\sqrt{x}}dx, \int_0^1 xe^{-x}dx.$$

Exercise 4 – Use the Riemann integral to calculate the limits

$$\lim_{n\to+\infty} \frac{1}{n}\sum_{i=1}^{n} \tan\left(\frac{i}{n}\right), \lim_{n\to+\infty} \sum_{i=1}^{n} \frac{i}{n^2+i^2}.$$

Exercise 5 – Calculate the area of the figure delimitated by

1. the parabolic $y = 4x - x^2$ and the x-axis.
2. the area between the curve of $y = \tan(x)$, the x-axis and the line $x = \frac{\pi}{3}$.
3. the area between the curve $y = 2 - x^2$, the curve $y = x^{\frac{2}{3}}$ and the x-axis.
