Level: LMD 3rd year – Applied Mathematics 2025/2026

Module: Advanced Probability

Exercise Sheet 2

Exercise 1

1) The MGF of X is $M_X(t) = \dfrac{1}{7-t^2}$. Find the MGF of $2X$.

2) Consider $M_X(t) = \exp(2e^t + 7t - 2)$. Compute the variance of X.

3) Let X be degenerate at 5. Find the MGF of X. Deduce the mean and variance.

4) A continuous random variable Y has PDF

$$f(y) = 3e^{-3y}, \quad y \geq 0.$$

Find the MGF of Y.

Exercise 2

1) Compute the MGF of a Binomial$(12, 0.6)$ random variable and deduce its mean and variance.

2) Show that the sum of n independent Bernoulli(p) random variables follows a Binomial(n, p) distribution using MGFs.

3) Compute the MGF of $X \sim \mathcal{N}(\mu, \sigma^2)$.

4) Compute the MGF of an Exponential random variable with rate λ.

Exercise 3

1. The value of a piece of factory equipment after three years of use is given by

$$V = 100(0.5)^X,$$

where X is a random variable whose MGF is

$$M_X(t) = \frac{1}{1-2t}, \quad t < \tfrac{1}{2}.$$

Compute the expected value of this equipment after three years of use.

2. For a random variable X, we know that

$$M_X(s) = \frac{2}{2-s}, \quad s \in (-2, 2).$$

Find the distribution of X.

Exercise 4

Let the random variable X have the probability density function

$$f(x) = \begin{cases} x, & 0 \le x \le 1, \\ 2 - x, & 1 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

a) Find the moment generating function (MGF) of X.

b) Verify that the MGF exists in a neighborhood of 0.

c) Using the MGF, find the mean and variance of X.

d) Deduce the characteristic function (CF) of X.

Exercise 5

1) Compute the characteristic function of $X \sim \mathcal{N}(\mu, \sigma^2)$.

2) Compute $\varphi_X(t)$ for $X \sim \mathcal{U}(0, 1)$.

3) Compute the characteristic function of an Exponential random variable with rate λ.

Exercise 6

1) Show that if $X \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $Y \sim \mathcal{N}(\mu_2, \sigma_2^2)$ are independent, then

$$X + Y \sim \mathcal{N}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2).$$

2) Find the characteristic function of a Poisson(λ) distribution and deduce its mean and variance.

3) Find the characteristic function of a Geometric(p) distribution and deduce its mean and variance.

4) Let $X_1, \ldots, X_n$ be independent random variables such that $X_i \sim \text{Poisson}(\lambda_i)$ for $i = 1, \ldots, n$. Using characteristic functions, show that

$$S = \sum_{i=1}^{n} X_i \sim \text{Poisson}\left(\sum_{i=1}^{n} \lambda_i\right).$$