Multiple-Choice Questions

Chapter 2: Moment Generating and Characteristic Functions

Q1. The moment generating function (MGF) of a random variable X is defined as:

a) $M_X(t) = \mathbb{E}[tX]$
b) $M_X(t) = \mathbb{E}[X^t]$
c) $M_X(t) = \mathbb{E}[e^{tX}]$
d) $M_X(t) = e^{\mathbb{E}[tX]}$

Q2. If the MGF $M_X(t)$ exists in a neighborhood of $t = 0$, then

a) All moments of X are infinite
b) The MGF uniquely determines the distribution of X
c) The distribution of X must be normal
d) X must be bounded

Q3. The first and second moments of X can be obtained from:

a) $M_X'(0)$ and $M_X''(0)$
b) $M_X(1)$ and $M_X(2)$
c) $M_X'(1)$ and $M_X''(1)$
d) $M_X(0)$ and $M_X'(0)$

Q4. If $X \sim \text{Exp}(\lambda)$, then its MGF is

a) $\dfrac{1}{1-\lambda t}$, $t < 1/\lambda$
b) $\dfrac{\lambda}{\lambda - t}$, $t < \lambda$
c) $\dfrac{t}{\lambda - t}$, $t < \lambda$
d) $e^{\lambda t}$

Q5. If $M_X(t) = e^{3t+2t^2}$, find $\mathbb{E}[X]$ and $\text{Var}(X)$.

a) 3, 4
b) 3, 2
c) 2, 3
d) 0, 2

Q6. If $M_X(t) = (0.7 + 0.3e^t)^{10}$, what is the distribution of X?

a) Poisson(3)
b) Binomial$(10, 0.3)$
c) Binomial$(10, 0.7)$
d) Geometric(0.3)

Q7. The characteristic function (CF) of X is defined as:

a) $\varphi_X(t) = \mathbb{E}[e^{tX}]$
b) $\varphi_X(t) = \mathbb{E}[e^{itX}]$
c) $\varphi_X(t) = e^{\mathbb{E}[itX]}$
d) $\varphi_X(t) = \mathbb{E}[it^X]$

Q8. The CF always exists for any random variable because:

a) $\mathbb{E}[|e^{itX}|] = 1$
b) $|e^{itX}| = |tX|$
c) $\mathbb{E}[e^{tX}] < \infty$
d) t is real

Q9. If $X \sim \mathcal{N}(0,1)$, then $\varphi_X(t) =$

a) $e^{-t^2/2}$
b) $e^{t^2/2}$
c) $e^{it^2/2}$
d) $\cos(t)$

Q10. If X and Y are independent, then the CF of $X + Y$ is:

a) $\varphi_{X+Y}(t) = \varphi_X(t) + \varphi_Y(t)$
b) $\varphi_{X+Y}(t) = \varphi_X(t)\varphi_Y(t)$
c) $\varphi_{X+Y}(t) = \varphi_X(t) + \varphi_Y(0)$
d) $\varphi_{X+Y}(t) = \mathbb{E}[XY]$

Q11. If $X \sim \text{Poisson}(\lambda)$, then its characteristic function is:

a) $e^{\lambda(e^{it}-1)}$
b) $e^{\lambda(it-1)}$
c) $\dfrac{\lambda}{\lambda - it}$
d) $e^{i\lambda t}$

Q12. The nth moment $\mathbb{E}[X^n]$ is obtained from:

a) $M_X^{(n)}(t)\big|_{t=0}$
b) $M_X^{(n)}(t)\big|_{t=1}$
c) $\dfrac{d^n M_X(t)}{dt^n}\Big|_{t=0}$
d) both (a) and (c)

Q13. If $M_X(t) = e^{5t}$, then X is:

a) Binomial$(5, 1)$
b) Poisson(5)
c) A degenerate random variable at 5
d) Normal$(5, 1)$